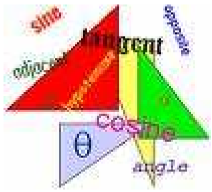
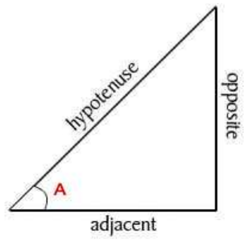


TRIGONOMETRY

Trigonometric ratios:



The six trigonometric ratios of A are:

Sine ***Cosine*** ***Tangent***

$$\sin A = \frac{opposite\ \ leg}{hypotenuse} \qquad \cos A = \frac{adjacent\ leg}{hypotenuse} \qquad \tan A = \frac{opposite\ \ leg}{adjacent\ leg}$$

and their inverses:

Cosecant ***Secant*** ***Cotangent***

$$\text{cosecant } A = \frac{1}{\sin \alpha} \qquad \text{secant } A = \frac{1}{\cos \alpha} \qquad \text{cotan } A = \frac{1}{\tan \alpha}$$

- ***Trigonometric rules:***

1. $\tan \alpha = \dfrac{\sin \alpha}{\cos \alpha}$

1. $\sin^2 \alpha + \cos^2 \alpha = 1$

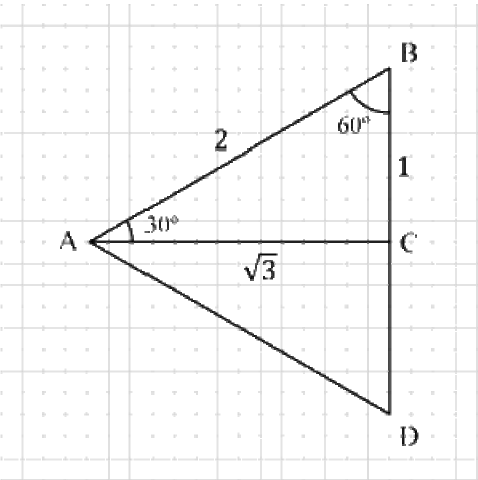
2. $\sec^2 \alpha = 1 + \tan^2 \alpha$

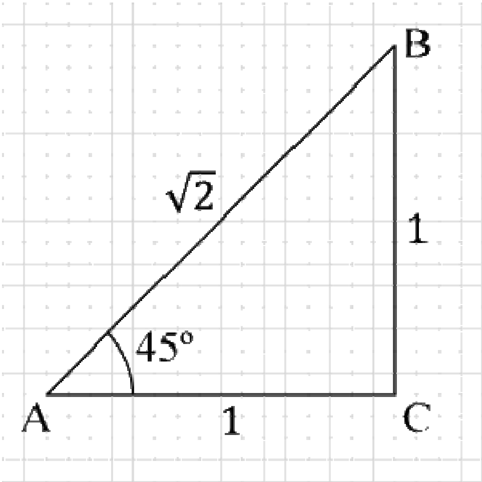
3. $\cos ec^2 \alpha = 1 + \cot an^2 \alpha$

Calculating trigonometric ratios

Use your scientific calculator to get the trigonometric ratios of angles, and be aware that the mode is DEG if the angle is expressed in degrees or RAD if it is expressed in radians.

You can calculate the exact value of the ratios for the angles of 30º, 45º and 60º. Use these triangles:





Learn by heart this is a table with the trigonometric ratios of 30º 45º and 60º:

	30º	45º	60º
sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Solving triangles:

(That means getting the measurements of all its sides and all its angles)

You really know a lot of facts about right triangles:

- Pythagorean theorem: $a^2 = b^2 + c^2$
- $A + B + C = 180^\circ$ or $B + C = 90^\circ$
- All the trigonometric ratios of angles B and C

C
a
b
B
A **c**

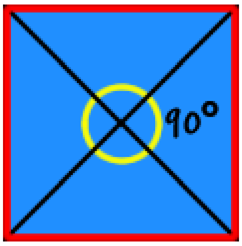
Remember!! Use in all the calculations the initial dates; each new measurement is an approximation, so, the error will increase if you use it.

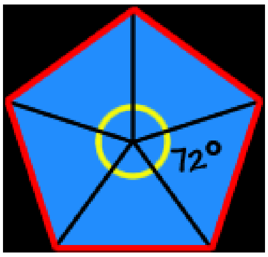
Calculations in geometry using angles:

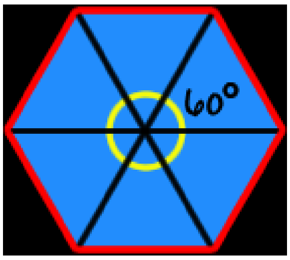
Measurements of angles and sides, apothems, radii and other elements of polygons are all related.

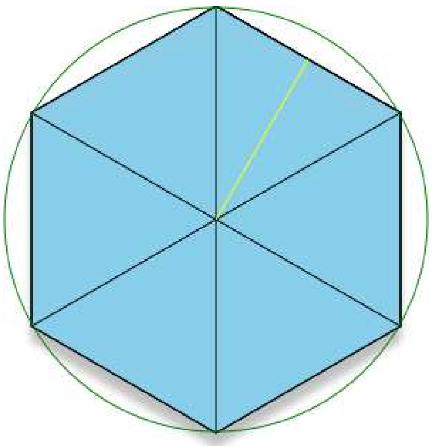
Use the trigonometric ratios to find the apothem and the radius if you know the side of a regular polygon.

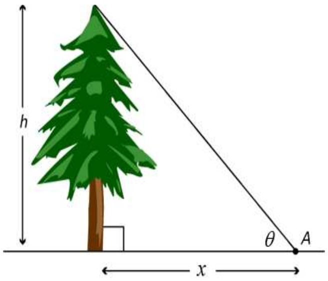
You can get the side and the apothem from the radius, etc...







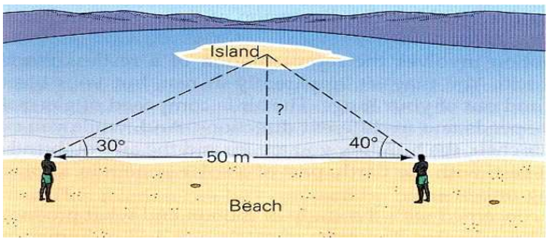




Applications of trigonometry:

Historically, Trigonometry was developed for astronomy and geography, but scientists have been using it for centuries for other purposes, too. Besides other fields of mathematics, trig is used in physics, engineering, and chemistry.

In the following examples we can find some of the uses of Trigonometry, as for example, getting the distance between the beach and the island in the picture below:

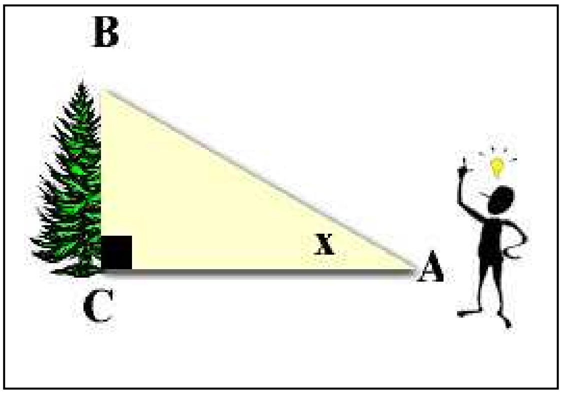


Remember these clues before solving a problem:

- If no diagram is given, draw one yourself.
- Mark the right angles in the diagram.
- Show the sizes of the other angles and the lengths of any lines that are known
- Mark the angles or sides you have to calculate.
- Consider whether you need to create right triangles by drawing extra lines. For example, divide an isosceles triangle into two congruent right triangles.
- Decide whether you will need Pythagoras theorem, sine, cosine or tangent.
- Check that your answer is reasonable. The hypotenuse is the longest side in a right triangle.

Before start, have a look at these definitions:

Angle of Elevation

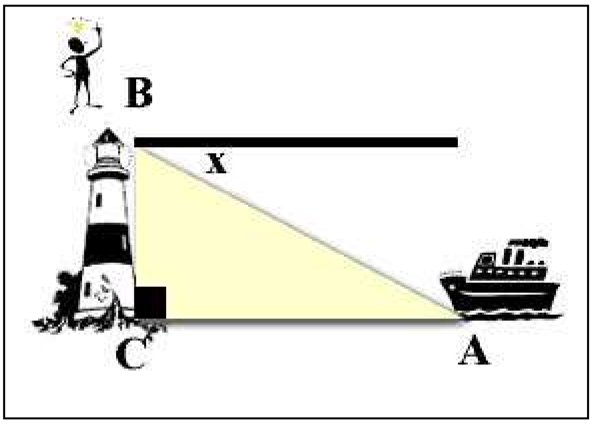


The *angle of elevation* is always measured from the ground up. Think of it like an elevator that only goes up. It is always **INSIDE** the triangle.

In the diagram at the left, ***x*** marks the angle of elevation of the top of the tree as seen from a point on the ground.

You can think of the angle of elevation in relation to the movement of your eyes. You are looking straight ahead and you must raise (elevate) your eyes to see the top of the tree.

Angle of Depression



The *angle of depression* is always **OUTSIDE** the triangle. It is never inside the triangle.

In the diagram at the left, ***x*** marks the angle of depression of a boat at sea from the top of a lighthouse.

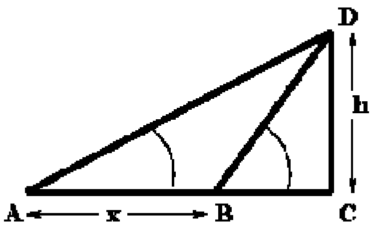
You can think of the angle of depression in relation to the movement of your eyes. You are standing at the top of the lighthouse and you are looking straight ahead. You must lower (depress) your eyes to see the boat in the water.

As seen in the diagram above of angle of depression, the dark black horizontal line is parallel to side CA of triangle ABC. This forms what are called alternate interior angles which are equal in measure (so, *x* also equals the measure of <*BAC*).
Simply stated, this means that:

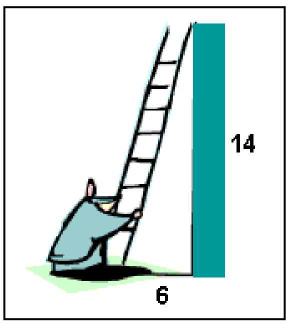
the angle of elevation = the angle of depression

Exercises and problems:

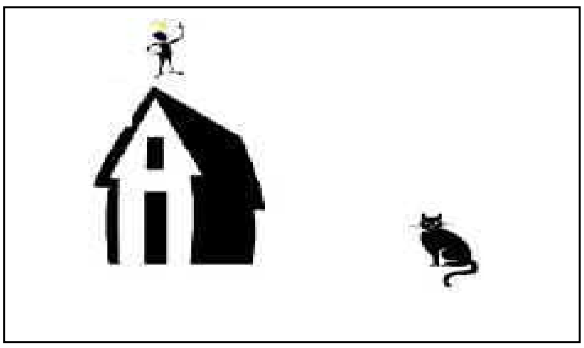
1. If the distance of a person from a tower is 100 m and the angle subtended by the top of the tower with the ground is 30^o, what is the height of the tower in meters?
2. A man is walking along a straight road. He notices the top of a tower subtending an angle $A = 60^o$ with the ground at the point where he is standing. If the height of the tower is $h = 35$ m, then what is the distance (in meters) of the man from the tower?
3. A little boy is flying a kite. The string of the kite makes an angle of 30^o with the ground. If the height of the kite is $h = 9$ m, find the length (in meters) of the string that the boy has used.
4. A ship of height $h = 24$ m is sighted from a lighthouse. From the top of the lighthouse, the angle of depression to the top of the mast and the base of the ship equal 30^o and 45^o respectively. How far is the ship from the lighthouse (in meters)?
5. Two men on opposite sides of a TV tower of height 32 m notice the angle of elevation of the top of this tower to be 45^o and 60^o respectively. Find the distance (in meters) between the two men.
6. Two men on the same side of a tall building notice the angle of elevation to the top of the building to be 30^o and 60^o respectively. If the height of the building is known to be h =100 m, find the distance (in meters) between the two men.



7. A pole of height h = 50 ft has a shadow of length l = 50.00 ft at a particular instant of time. Find the angle of elevation (in degrees) of the sun at this point of time.
8. A plane is approaching your home, and you assume that it is traveling at approximately 550 miles per hour.
   If the angle of elevation of the plane is 16 degrees at one time and one minute later the angle is 57 degrees, approximate the altitude. (We assume that the altitude is constant)
9. A ladder 6 feet long leans against a wall and makes an angle of 71º with the ground. Find to the *nearest tenth* of a foot how high up the wall the ladder will reach.
10. A ladder leans against a building. The foot of the ladder is 6 feet from the building. The ladder reaches a height of 14 feet on the building. Find the angle between the ladder and the building.

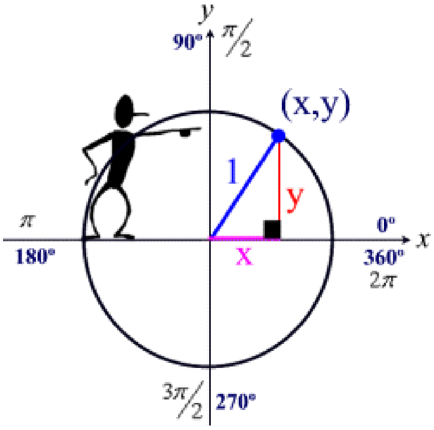


11. From the top of a barn 25 feet tall, you see a cat on the ground. The angle of depression of the cat is 40º. How many feet, to the *nearest foot,* must the cat walk to reach the barn?



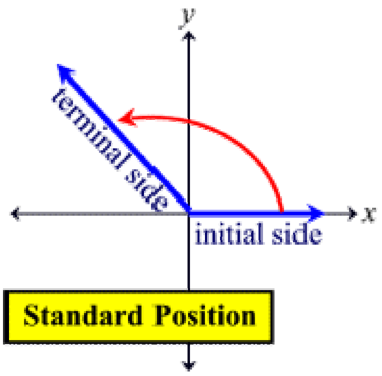
Unit circle:

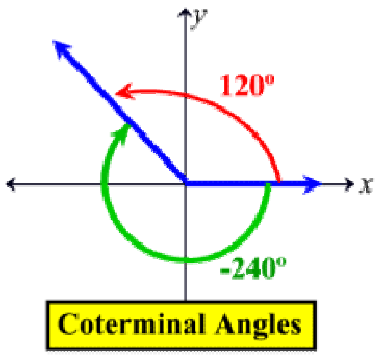
In Trigonometry, a unit circle is a circle with radius of 1 that is centred in the origin of co-ordinates.



In the unit circle, we can draw positive angles, negative angles and even angles bigger than 360°. If measured in a counter clockwise direction the measurement is positive. If measured in a clockwise direction the measurement is negative.

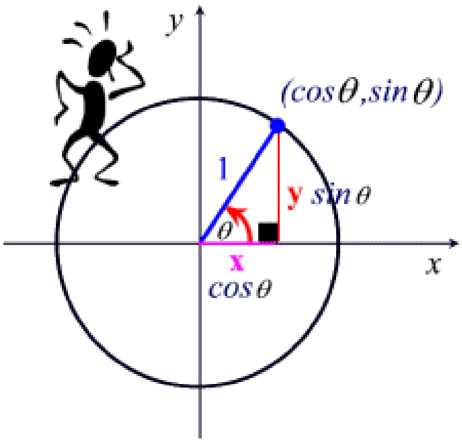
The standard position of an angle and how to draw negative angles are shown in the pictures below:



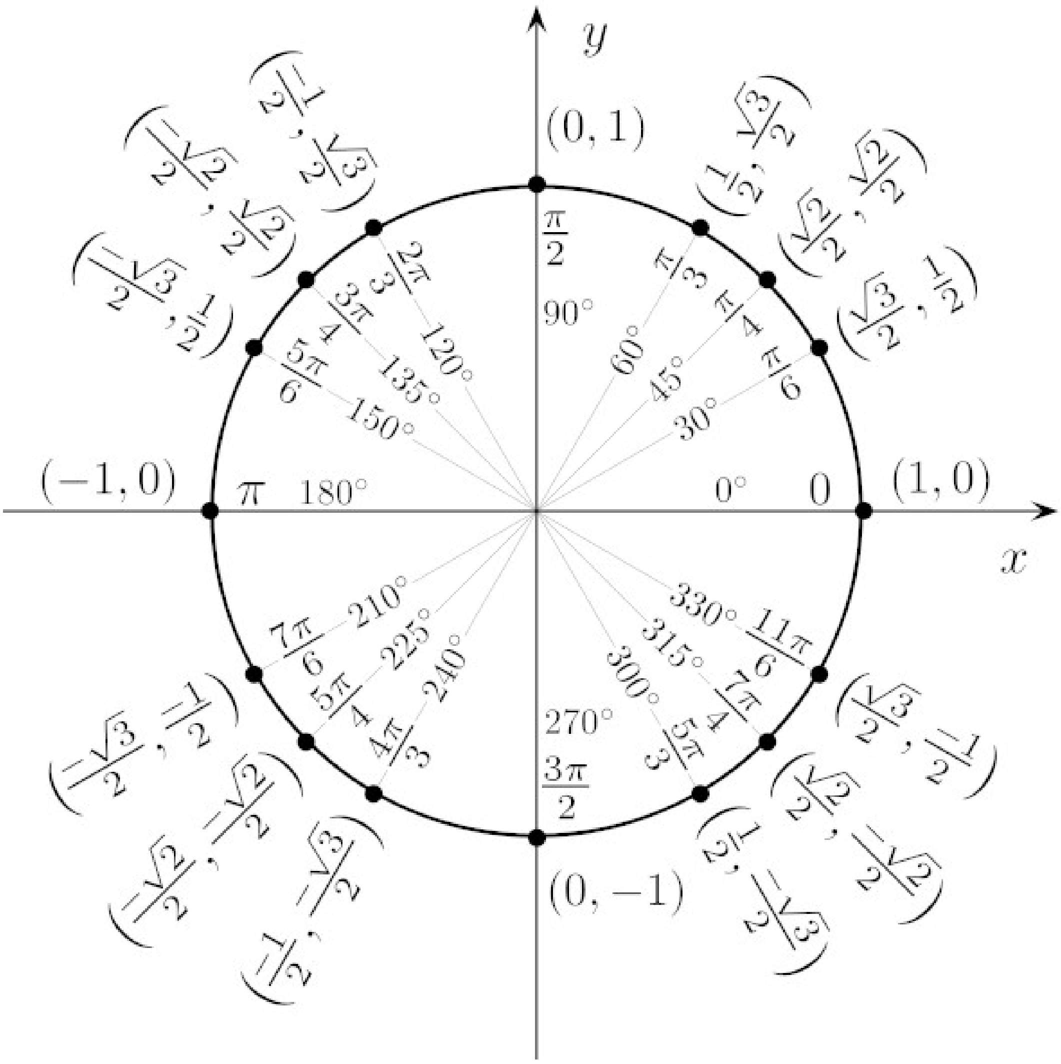


Trigonometric ratios:

We define the trigonometric ratios of a given angle θ using the co-ordinates of the point that the angle determines on the unit circle: cosθ = x, sinθ = y



Points of Special Interest on the Unit Circle:



Sign of the trigonometric ratios:

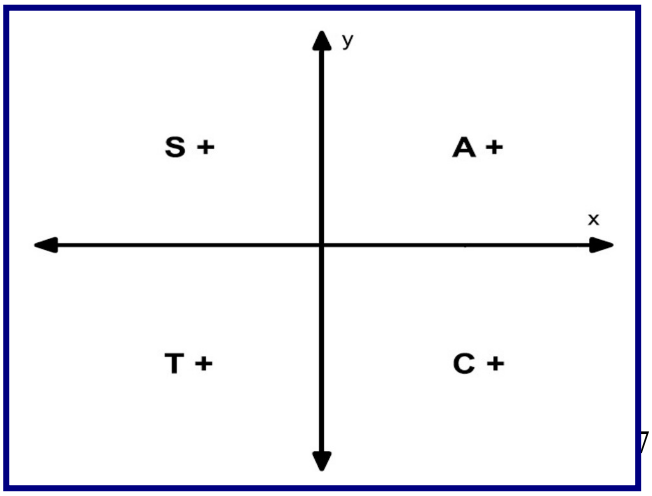
(The CAST diagram)

C: Cosine positive

A: All ratios positive

S: Sine positive

T: Tangent positive



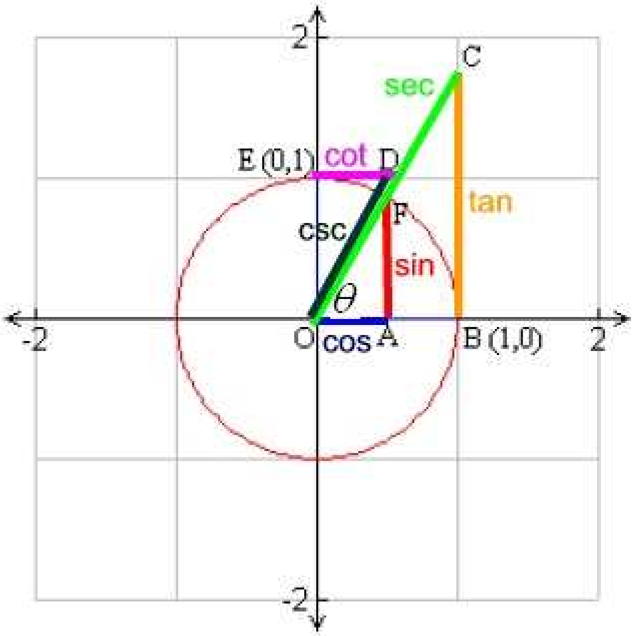
Segments which represent the trigonometric ratios:

Each of the six trig functions can be thought of as a length related to the unit circle, in a manner similar to that seen in the picture.

We have seen that the sine (**AF**) and cosine (**OA**) functions are distances from a point on the unit circle to the axes.

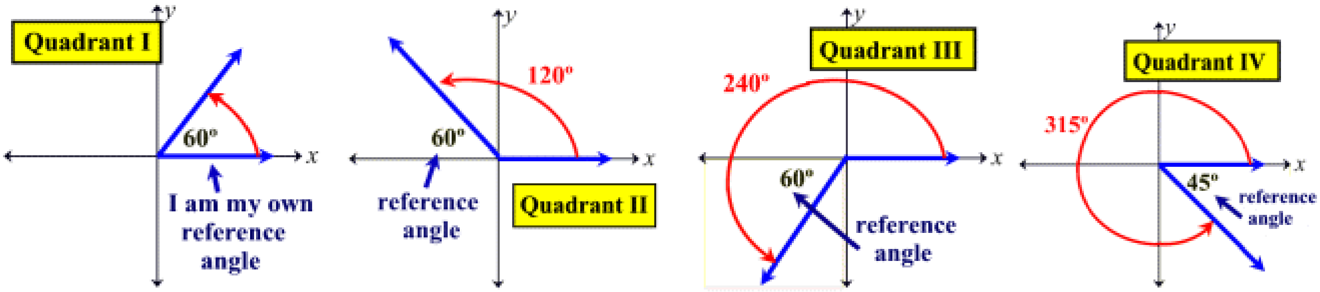
The tangent (**BC**) and cotangent (**ED**) functions are the lengths of the line segments tangent to the unit circle from the axis to the terminal ray of angle θ.

The secant (**OC**) and cosecant (**OD**) functions are the lengths on the rays (or secant lines), from the origin to its intersection with the tangent lines



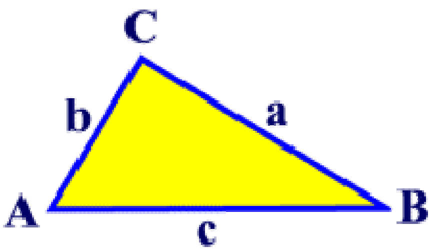
Reference angles:

Associated with every angle drawn in standard position there is another angle called the reference angle. The reference angle is the acute angle formed by the terminal side of the given angle and the x-axis. Reference angles may appear in all four quadrants. Angles in quadrant I are their own reference angles.



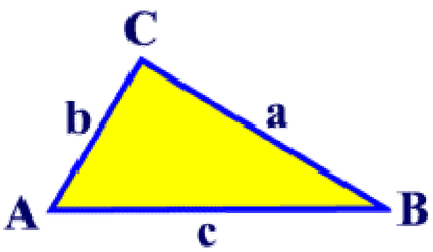
An *oblique* triangle is any triangle that is not a right triangle. It could be an acute triangle (all three angles of the triangle are less than right angles) or it could be an obtuse triangle (one of the three angles is greater than a right angle). Actually, for the purposes of trigonometry, the class of "oblique triangles" might just as well include right triangles, too. Then the study of oblique triangles is really the study of all triangles. Solve oblique triangles using these two rules:

Law of sines:



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of cosines:



$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

You may use the law of sines or the law of cosines depending on the initial data. Try these exercises:

1. *A* and *B* are points on opposite sides of a river. On one bank the line *AC* 650 feet is measured. The angle *A* = 73° 40', and *C* = 52° 38'. Find *AB.*
2. The sides of a parallelogram are *AB* = 209.16 and *AD* =347.25, and the diagonal *AC* = 351.47. Find the angles and the other diagonal.
3. *AB* is a line 652 feet long on one bank of a stream, and *C* is a point on the opposite bank. *A* = 53° 18', and *B* = 48° 36'. Find the width of the stream from *C* to *AB.*
4. The diagonals of a parallelogram are 374.14 and 427.21 and the included angle is 70° 12' 38". Find the sides.

Your own UNIT CIRCLE

The unit circle (where the radius equals 1) provides a very clear demonstration of how various trigonometric functions relate to angles and one another. Draw an angle line from the origin to a point on the circumference of the circle; the (x,y) coordinates of that point will be the cosine and sine of the angle.